a known temperature. Subscripts: 1, initial binder; 2, 3, intermediate and condensed pyrolysis products; 4, inert filler; 5, gas phase; α, number of the gaseous component; H, initial state; K, final state; w, surface $Y_1(t)$. IIM, iteration-interpolation method.

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SOLUTION OF A TWO-DIMENSIONAL HEAT-CONDUCTION PROBLEM FOR A GEOMETRICALLY COMPLEX DOMAIN BY AN INTEGROINTERPOLATION METHOD

N. V. Kerov

UDC 536.24

A methodology is proposed for the construction of an algorithm to solve heat transfer problems for spatial domains of complex geometric shape.

The creation of a new engineering operating under high-temperature loading or intensive cooling conditions is associated with carrying out a large amount of special temperature investigations of materials and structures. Such operations are a constant necessity for many branches of industry, consequently, thermal computation methods are also perfected simultaneously with the rise in the demands on engineering systems. Computational algorithms based on one-dimensional formulations of heat transfer problems are most widespread. If a notable fraction of algorithms arrived earlier at analytic methods of solution, then numerical methods have acquired greater weight at this time in connection with the development of computer technology. These methods possess a substantial advantage resulting from the possibility of their utilization for different formulations of problems, for instance, with any nonlinearities taken into account.

However, when studying fine physical processes associated with structure heating, it is already not always sufficient to utilize a one-dimensional heating model Hence, a large quantity of researches has appeared devoted to methods and algorithms for the solution of heat transfer problems in multidimensional formulations.

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All the above is valid even for inverse heat transfer problems. It should be noted that by far still not all the domains of possible application are included in the domain of solving inverse problems by one-dimensional formulations. As materials of the All-Union Seminar on Inverse Problems held in 1987 show, the most unexpected formulations are possible. However, even here the study of temperature processes most often requires the passage from one- to multidimensional formulations of inverse heat transfer problems. For instance, to determine the heat flux distributed over the external surface of a structure that possesses high heat conduction along the generator, a one-dimensional heat conduction model or, say, a certain heat flux sensor inserted in the structure and using the one-dimensional heat propagation model cannot always be utilized. In these cases the use of inverse problems in multidimensional formulations [1] may be the single exit from the situation. The solution of inverse problems on the basis of two-dimensional, and even more, of three-dimensional heat propagation models can cause definite difficulties. The purely mathematical questions associated, say, with uniqueness of the solutions should here be extracted. Purely computational difficulties also exist that appear in connection with the limited fast-response and operational storage of modern computers.

Relying on the above, we make the deduction that it is necessary to have a good algorithm (program) for the solution of the appropriate direct heat transfer problem in order to develop an effective algorithm of the solution of inverse heat transfer problems in particular. What should be understood as a good algorithm will be clear from the subsequent text.

An algorithm of the solution of the direct heat transfer problem for a generalized two-dimensional equation is examined in this paper.

The creation of algorithms for direct heat conduction problems does not, as a rule, cause difficulties in principle. However, the algorithms can differ substantially from each other in specific formulations. This is related to the manifold of two-dimensional problems encountered in temperature investigation practice. The problems can differ in the stage of examining the spatial domain in which the solution will be sought. The domain can be a simple rectangular shape. The heat transfer equation in this case is written in a rectangular coordinate system. In more complex cases one of two possible planes in a cylindrical or spherical coordinate system must be used. In practice, simple geometric shapes are encountered much more rarely than complex shapes having, say, curved boundaries that do not fit into an orthogonal grid. Moreover, the computed geometric domain can itself be multiconnected, have "vacancies" or inclusions characterized by singular thermal conditions or thermophysical characteristics. It is visibly impossible to propose a certain generalizing approach that would assure the creation of an absolutely universal algorithm capable of solving the heat transfer problem for any of the designated cases. However, preparation of the algorithms for specific formulations would result in the creation of a large quantity of separate algorithms and programs that can, as a rule, be used only by the authors themselves.

An analogous situation occurs even in the determination of sets of initial data.

Let us examine the influence of specific geometric shapes on algorithms of heat transfer problems in more detail.

An interesting method used in problems to compute the flow around complex bodies is proposed in [2]. The method maps the physical flow domain into a rectangular domain on a plane. In other words, the solution of the partial differential equations is executed in curvilinear coordinate systems consistent with the boundaries of an arbitrary domain. Special slits along the connection boundaries are introduced for multiconnected domains during the mapping onto a single rectangle. However, sufficiently complex transformations must be performed to obtain the difference analogs of the initial equations. Initially the derivatives in Cartesian coordinates are converted into differential expressions with derivatives in curvilinear coordinates and derivatives of the Cartesian coordinates with respect to the curvilinear. Only afterwards is the difference approximation of the derivatives realized in the transformed domain. Approximation errors are estimated for the transformations performed. The method is presented in this paper as an example of the possible approaches and, consequently, questions associated with the approximation errors of this method are not examined. However, the investigations performed show that for large deviations of the initial curvilinear grid from orthogonality, the approximation errors increase especially in the near-body domain where one-sided derivatives are utilized as a rule [2]. The method of constructing adaptive

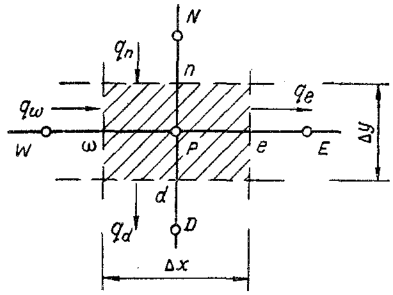

Fig. 1. Control volume.

grids [3] should be considered another direction. Its sense is that an adaptive displacement of the nodes is performed along one coordinate for the grid constructed by some method so as to achieve the best possible conditions for solving the problem.

It is seen from all the above that a great deal of attention is expended on questions of discretization of problems in the computational domains in thermo- and hydrodynamics.

In the case when a computational grid has been selected successfully, the question of selecting the numerical method for solving the problem occurs.

Algorithms utilizing complex discrete representations of the initial differential equations are not touched upon in this paper since these algorithms have a sufficiently limited use for special classes of problems.

The presence of a large quantity of physical problems related to heat transfer and hydrodynamics processes resulted in the growth of a quantity of programs utilizing substantially the same algorithm for the solution of the direct problem on the basis of a generalized heat transfer equation. These programs are differentiated by structure organization and by methods of giving the initial information. In this case the desire to create a sufficiently universal program (complex of programs), capable of being adapted easily to different formulations of the direct heat conduction problems and consequently to inverse problem formulations also, is completely natural. The program complex to solve inverse heat transfer problems in one-dimensional formulations, as proposed in [6, 7], can be called the prototype of such a program. It should be noted that this complex is the most complete at this time. It is easily adapted to the solution of diverse problems of heat transfer: direct, inverse boundary, inverse coefficient, and optimal temperature measurement planning problems.

Many approaches utilized in this complex remain legitimate in going over to problems in two-dimensional formulations. But the very problem of creating a universal complex is sufficiently complicated since the amount of needed initial information grows multiply. Program adaptation to a set of geometric shapes, to a set of possible boundary conditions must be provided for here. In the case of multiconnected computational domains, the program should use different thermophysical coefficients, including the anisotropic (orthotropic).

Let us examine one of the possible specific realizations of such a program. We first select the integrointerpolation method proposed in [4] as the numerical method. This method possesses good "physicality." Its modification is developed for the case of a generalized equation and is designated the "control volume" method in [8]. It should be stipulated at once that despite the fact that the selected method imposes its imprint on the formation of the algorithm and program, another numerical method, the method of varible directions, say, can be realized in the program. Such a substitution does not cause any substantial changes in the program structure.

Let us examine the numerical solution method. A small volume separated in the computational domain in which energy balance conditions are assured is represented in Fig. 1. Within the limits of this volume a generalized heat transfer equation of the following kind is valid:

$$\rho C(T)\frac{\partial T}{\partial \tau}+\rho\frac{u_x}{D_r}\frac{\partial T}{\partial x}+\rho u_y\frac{\partial T}{\partial y}=B_r\frac{\partial}{\partial x}\left(\lambda_x(T)\frac{\partial T}{\partial y}\right)+\frac{1}{A_r}\frac{\partial}{\partial y}\left(\lambda_y(T)A_r\frac{\partial T}{\partial y}\right)+q_V(T), \qquad (1)$$

where $A_r = 1$, $B_r = 1$, $D_r = 1$ for a rectangular coordinate system, $x = \varphi$, $y = r$, $A_r = r$, $B_r = 1/r^2$, $D_r = r$ for a cylindrical coordinate system (radical section), $x = z$, $y = r$, $A_r = r$, $B_r = 1$, $D_r = 1$ for a cylindrical coordinate system (axial section), ρu_x, ρu_y are the convection rates, respectively, in the Ox and Oy directions, $\rho C(T)$ is the volume specific heat, $\lambda_x(T)$, $\lambda_y(T)$ are the heat conductions, respectively, in the Ox and Oy directions, and $q_V(T)$ is a volume source or sink.

To obtain the discrete analog of (1), we integrate this equation in a small volume while simultaneously taking the average within the limits of a finite increment $\Delta\tau$. We represent the first derivatives in the expression obtained in finite-difference form. Furthermore, following [4], we select an implicit approximation scheme of finite-difference expressions for the derivatives, resulting in the following mode of writing the discrete analog of (1)

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_D T_D + b, \tag{2}$$

where

$$a_E = -\frac{\rho u_e}{2} S_e + \frac{\lambda_e}{(\Delta x)_e}\frac{S_e}{D_P};\quad a_W = \frac{\rho u_\omega}{2} S_e + \frac{\lambda_\omega}{(\Delta x)_\omega}\frac{S_e}{D_P};$$

$$a_N = -\frac{\rho u_n}{2} S_n + \frac{\lambda_n}{(\Delta y)_n} S_n;\quad a_D = \frac{\rho u_d}{2} S_d + \frac{\lambda_d}{(\Delta y)_d} S_d;$$

$$a_P = \frac{\rho u_n}{2} S_n - \frac{\rho u_d}{2} S_d + \frac{\rho u_e}{2} S_e - \frac{\rho u_\omega}{2} S_e + \frac{\lambda_n}{(\Delta y)_n} S_n + \frac{\lambda_d}{(\Delta y)_d} S_d + \frac{\lambda_e}{(\Delta x)_e}\frac{S_e}{D_P} + \frac{\lambda_\omega}{(\Delta x)_\omega}\frac{S_e}{D_P} + \frac{\rho C_P}{\Delta\tau} V_P - K_P V_P;$$

$$b = K_e V_P + \frac{\rho C_P}{\Delta\tau} V_P T^\circ.$$

Entering into the discrete analog (2) of the initial differential equation are S_e, S_n, and S_d, the areas of the control volume faces, λ_e, λ_w, λ_n, λ_d, the heat conduction of the faces, u_e, u_w, u_n, u_d are the rates of convection through the faces, T° is the temperature at the preceding time, and K_c, K_p are the coefficients of the linearized source.

Using (2), a discrete grid can be constructed whose nodes are arranged at the centers of the control volumes. If a condition is posed that the faces of the near-boundary control volumes agree with the boundaries of the spatial domain, then in this case the boundary conditions can be introduced as coefficients of the linearized source.

Without taking convection into account the boundary conditions can be written as follows in general form

$$K_1 \frac{\partial T}{\partial n} + K_2 T + K_3 = 0, \tag{3}$$

where $K_1 = 0$, $K_2 = 1$, $K_3 = -T_B$ are boundary conditions of the first kind; $K_1 = \lambda$, $K_2 = 0$, $K_3 = -q_B$ are boundary conditions of the second kind; $K_1 = \lambda$, $K_2 = \alpha$, $K_3 = -\alpha T_m$ are boundary conditions of the third kind, T_B, q_B are the temperature and heat flux on the boundary, α is the heat transmission coefficient, and T_m is the temperature of the medium from outside the boundary.

In this case the coefficients of the linearized source for boundary conditions of the second kind, say, are represented as

$$K_P = 0;\ K_C = q_B S_B, \tag{4}$$

where S_B is the area of the control volume face that agrees with the boundary of the spatial domain.

The expressions (3) and (4) are valid if and only if the face of the control volume agrees with the boundary of the spatial domain. But another case is possible, when the nodes of the discrete grid formed agree with the boundaries. Here, (1) must be integrated in half the control volume in order to approximate the control volume.

The considered formulation of the problem is valid even for multiconnected geometrically complex domains in a rectangular or cylindrical coordinate system. If straight lines are drawn parallel to the coordinate system directions along breaks of the outer boundary (Fig. 2), then the constructed enlarged grid forms new subdomains. We later call these subdomains macrovolumes. Such an approach is used in [6, 7]. The formulation (1) and (3) of the heat transfer problem is valid within the macrovolumes. The factorization method can be used for the solution along the lines of the discrete grid.

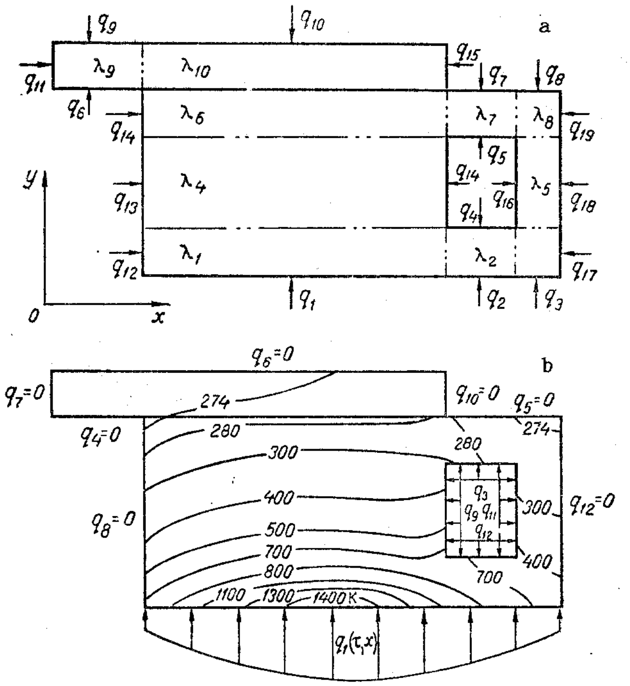

Fig. 2. Determination of the two-dimensional temperature field in a multiconnected domain: a) partition of the multiconnected domain into macrovolumes and possible assignment of the initial data (boundary conditions and thermophysical coefficients); b) example of temperature field determination.

Let us consider the heat transfer condition on the macrovolume boundaries. The boundary conditions on the boundary common with the spatial domain are approximated analogously to (3). The conditions on the connecting boundaries in the case of ideal contact are satisfied automatically, i.e., the solution of the problem along the discrete grid lines is realized by continuous factorization. A special approximation of the conditions on the connecting boundary must be realized in the presence of a contact (thermal) resistance. For instance, the expression [6]

$$K_4\lambda(T)\frac{\partial T_i}{\partial x}+K_5T_i-K_6\lambda(T)\frac{\partial T_{i+1}}{\partial x}-K_7T_{i+1}=K_8,$$
$$K_9\lambda(T)\frac{\partial T_i}{\partial x}+K_{10}T_i-K_{11}T_{i+1}=K_{12} \qquad (5)$$

can be written for the conditions on the connecting boundary in the Ox axis direction (Fig. 3), where K_j, $j=\overline{4,12}$ are coefficients governing the specific energetic conditions on the connecting boundary. In the general case K_j can depend on the time. If (1) is integrated in the control volume halves located around the discrete grid nodes, then a discrete analog of the type

$$a_{B_1}T_{B_1}=a_WT_W+a_ET_E+b \qquad (6)$$

is obtained after finite-difference approximation of the first derivatives, where

$$a_W=\left(K_4K_{11}+K_9\frac{M_{B_2}}{S_{B_1}}\right)\left\{D_\omega\left[\left|0,\left(1-\frac{0,1|U_\omega|}{D_\omega}\right)^5\right|\right]+[|U_\omega,0|]\right\};$$

$$a_E=K_{11}K_6\left\{D_e\left[\left|0,\left(1-\frac{0,1U_e}{D_e}\right)^5\right|\right]+[|-U_e,0|]\right\};$$

$$a_{B_1}=a_W+K_4K_{11}(U_{B_1}-U_\omega)+K_9\frac{M_{B_2}}{S_{B_1}}(U_{B_1}-U_\omega)+$$

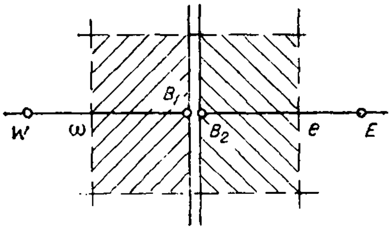

Fig. 3. Control volumes at the boundary of the energy connection of the subdomains.

$$+ (K_{11}K_5 + K_{10}M_{B_2}) + \left(K_4K_{11} + K_9\frac{M_{B_2}}{S_{B_1}}\right)\left(\frac{C_{B_1}V_{B_1}}{\Delta\tau} - K_{B_1}V_{B_1}\right);$$

$$b = K_{11}K_3 + K_{12}M_{B_2} + \left(K_4K_{11} + K_9\frac{M_{B_2}}{S_{B_1}}\right)\left(\frac{C_{B_1}V_{B_1}}{\Delta\tau}T^0_{B_1} + K_{C_1}V_{B_1}\right) + K_{11}K_6\left(\frac{C_{B_2}V_{B_2}}{\Delta\tau}T_{B_2} - K_{C_2}V_{B_2}\right);$$

$$U_\omega = S_\omega\rho u_\omega,\ U_{B_1} = S_{B_1}\rho u_{B_1},\ U_e = S_e\rho u_e,$$

$$D_e = \frac{S_e\lambda_e}{(\Delta x)_e},\quad D_\omega = \frac{S_\omega\lambda_\omega}{(\Delta x)_\omega}.$$

The quantity M_{B_2} in (6) governs the heat transfer condition from the node B_2, S_{B_1} is the area of the control volume faces making contact, and ρu_{B_1} is the convection rate through the faces making contact.

The operator $[|f_1, f_2|]$ is introduced in [8] and denotes selection of the greatest values of f_1 and f_2.

Therefore, by using the discrete analogs (2) and (6) and the methodology of forming sets of macrovolumes, the heat transfer problem can be solved for different spatial domains of geometrically complex shape. Moreover, intrinsic boundary conditions and thermophysical coefficients can be given for each macrovolume, which affords the possibility of using different sets of initial and boundary conditions for the solution of the problem.

An algorithm is compiled on the basis of the methodology considered and a program is written for the solution of heat transfer problems. The possibility of rapid adaptation of the program for spatial domains of different geometric shape and for different sets of boundary conditions and thermophysical coefficients is provided.

In conclusion, it should be noted that approximation of the macrovolume faces must be considered for spatial domains with curvilinear boundaries. Step boundaries or triangular control volumes adjoining the curvilinear macrovolume boundaries can be used for this.

The temperature field obtained by using the proposed algorithm is represented in Fig. 2b as an illustration.

NOTATION

T, temperature, τ, time; Oxy, rectangular coordinate system; ρ, density; C, specific heat; λ, heat conduction; u, convection rate; O_r, cylindrical coordiante system; A_r, B_r, D_r, coefficients connecting the coordinate systems; q, a source; a_P, a_E, a_W, a_N, a_D, coefficients of the difference analog; S, area of a control volume face; V_P, control volume; T^0, temperature of the previous time layer; K_P, K_C, linearized source coefficients; K_1, K_2, K_3, coefficients of a generalized writing of the boundary conditions; K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, coefficients of a generalized writing of the energy coupling conditions.

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CONSTRUCTION OF SMOOTHING SPLINES BY LINEAR PROGRAMMING METHODS

A. G. Pogorelov

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The mathematical questions and algorithms for constructing n-th order smoothing splines by means of experimental (kinetic) dependences are elucidated.

1. Let the function $f(x) \in C^Q[X]$, $Q \geq n$ that takes on the approximate values $f(x_1) + \delta_1, \ldots, f(x_N) + \delta_N$ be given discretely with the errors $\delta_1, \ldots, \delta_N$ at the nodes $x_1, \ldots, x_N$ on the segment $X \subset R$. It is required to approximate the function f(x) in each interval $[x_i, x_{i+1})$, $i = \overline{1, N-1}$ by a polynomial of n-th degree, $n \geq 3$:

$$y_i(x) = a_{0i} + a_{1i}x + a_{2i}x^2 + \cdots + a_{ni}x^n, \; x \in [x_i, x_{i+1}) \tag{1}$$

so as to satisfy the requirements [1-6]: I) fusion of the spline derivatives at the mesh nodes $S = \{x_1, \ldots, x_N\}$ up to the (n − 1) order

$$\begin{cases} a_{0i} + a_{1i}x_i + a_{2i}x_i^2 + \cdots + a_{ni}x_i^n = a_{0,i+1}, \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ (n-1)!\, a_{n-1,i} + n!\, a_{ni}x_i = a_{n-1,i+1}, \; i = \overline{1, N-2}; \end{cases} \tag{2}$$

II) the requirement of minimal variation of the (n − 1)-derivative of $y_i(x)$ (i.e., $\int\limits_{x_1}^{x_N} (y^{(n-1)}(x))^2 dx \underset{a}{\to} \min$), corresponding to condition $|a_{\nu i}| \underset{a}{\to} \min$, $\nu = n-1, n$, $i = \overline{1, N-1}$, in order to avoid oscillating behavior of the graph of the spline; III) location of the spline graph within the error corridor:

$$\begin{cases} |f_\delta(x_i) - a_{0i}| \leq \delta_i, \; i = \overline{1, N-1}, \\ |f_\delta(x_N) - a_{0,N-1} - a_{1,N-1}x_N - \cdots - a_{n,N-1}x_N^n| \leq \delta_N. \end{cases} \tag{3}$$

2. Conditions I and III yield the search domain for the interval values of the spline approximation coefficients by the system of constraints

$$\begin{cases} a_{0i} \leq f_\delta(x_i) + \delta_i, \\ -a_{0i} \leq -f_\delta(x_i) + \delta_i, \\ a_{0i} + a_{1i}x_{i+1} + a_{2i}x_{i+1}^2 + \cdots + a_{ni}x_{i+1}^n - a_{0,i+1} = 0, \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ (n-1)!\, a_{n-1,i} + n!\, a_{ni}x_{i+1} - a_{n-1,i+1} = 0, \; i = \overline{1, N-2}, \\ a_{0,N-1} \leq f_\delta(x_{N-1}) + \delta_{N-1}, \end{cases} \tag{4}$$

N. D. Zelinskii Institute of Organic Chemistry, Academy of Sciences of the USSR, Moscow.
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